Mathematics
Paper 3 (Calculator)

Higher Tier

You must have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser, calculator.

Instructions
• Use black ink or ball-point pen.
• Fill in the boxes at the top of this page with your name, centre number and candidate number.
• Answer all questions.
• Answer the questions in the spaces provided – there may be more space than you need.
• Calculators may be used.
• If your calculator does not have a π button, take the value of π to be 3.142 unless the question instructs otherwise.
• Diagrams are NOT accurately drawn, unless otherwise indicated.
• You must show all your working out.

Information
• The total mark for this paper is 80
• The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.

Advice
• Read each question carefully before you start to answer it.
• Keep an eye on the time.
• Try to answer every question.
• Check your answers if you have time at the end.
Answer ALL questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1. The ratio of the number of boys to the number of girls in a school is 4:5
   There are 95 girls in the school.
   Work out the total number of students in the school.

(Total for Question 1 is 3 marks)

2. The diagram represents a solid made from seven centimetre cubes.

   On the centimetre grid below, draw a plan of the solid.

(Total for Question 2 is 2 marks)
3 Make $t$ the subject of the formula $y = \frac{t}{3} - 2a$

(Total for Question 3 is 2 marks)

4 Jim rounds a number, $x$, to one decimal place.
The result is 7.2
Write down the error interval for $x$.

(Total for Question 4 is 2 marks)
5 Katie measured the length and the width of each of 10 pine cones from the same tree.

She used her results to draw this scatter graph.

(a) Describe one improvement Katie can make to her scatter graph.

The point representing the results for one of the pine cones is an outlier.

(b) Explain how the results for this pine cone differ from the results for the other pine cones.

(Total for Question 5 is 2 marks)
At a depth of \( x \) metres, the temperature of the water in an ocean is \( T \)°C.
At depths below 900 metres, \( T \) is inversely proportional to \( x \).

\[
T = \frac{4500}{x}
\]

(a) Work out the difference in the temperature of the water at a depth of 1200 metres and the temperature of the water at a depth of 2500 metres.

\[\text{..................................................................................................................................................................................................................................................}°\text{C}\]

(3)

Here are four graphs.

One of the graphs could show that \( T \) is inversely proportional to \( x \).

(b) Write down the letter of this graph.

\[\text{..................................................................................................................................................................................................................................................}\]

(1)

(Total for Question 6 is 4 marks)
7 Here is a right-angled triangle.

Four of these triangles are joined to enclose the square $ABCD$ as shown below.

Show that the area of the square $ABCD$ is $x^2 + y^2$

(Total for Question 7 is 3 marks)
8 The diagram shows an oil tank in the shape of a prism. The cross section of the prism is a trapezium.

The tank is empty.

Oil flows into the tank.
After one minute there are 300 litres of oil in the tank.

Assume that oil continues to flow into the tank at this rate.

(a) Work out how many **more** minutes it takes for the tank to be 85% full of oil.

\[1 \text{ m}^3 = 1000 \text{ litres}\]

...................................................... minutes

(5)

(b) Explain how this could affect your answer to part (a).

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(1)

(Total for Question 8 is 6 marks)
9 Ibrar bought a house for £145 000

The value of the house depreciated by 4% in the first year.
The value of the house depreciated by 2.5% in the second year.

Ibrar says,

“4 + 2.5 = 6.5 so in two years the value of my house depreciated by 6.5%”

(a) Is Ibrar right?
You must give a reason for your answer.

The value of Ibrar’s house increases by $x\%$ in the third year.
At the end of the third year the value of Ibrar’s house is £140 000

(b) Work out the value of $x$.
Give your answer correct to 3 significant figures.
10 The surface gravity of a planet can be worked out using the formula

\[ g = \frac{6.67 \times 10^{-11}}{r^2} \]

where

- \( m \) kilograms is the mass of the planet
- \( r \) metres is the radius of the planet

For the Earth and Jupiter here are the values of \( m \) and \( r \).

<table>
<thead>
<tr>
<th></th>
<th>Earth</th>
<th>Jupiter</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m )</td>
<td>( 5.98 \times 10^{24} )</td>
<td>( 1.90 \times 10^{27} )</td>
</tr>
<tr>
<td>( r )</td>
<td>( 6.378 \times 10^6 )</td>
<td>( 7.149 \times 10^7 )</td>
</tr>
</tbody>
</table>

Work out the ratio of the surface gravity of Earth to the surface gravity of Jupiter. Write your answer in the form 1: \( n \)

(Total for Question 10 is 3 marks)
11 Solve the simultaneous equations

\[\begin{align*}
2x - 4y &= 19 \\
3x + 5y &= 1
\end{align*}\]

\[x = \ldots \]
\[y = \ldots \]

(Total for Question 11 is 4 marks)
12  Zahra mixes 150 g of metal A and 150 g of metal B to make 300 g of an alloy.

   Metal A has a density of 19.3 g/cm³.
   Metal B has a density of 8.9 g/cm³.

   Work out the density of the alloy.

   .......................................................  g/cm³

   (Total for Question 12 is 4 marks)
On the grid, enlarge the triangle by scale factor $-1\frac{1}{2}$, centre (0, 2)

(Total for Question 13 is 2 marks)
The table gives information about the speeds, in km/h, of 81 cars.

<table>
<thead>
<tr>
<th>Speed (s km/h)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>90 &lt; s ≤ 100</td>
<td>13</td>
</tr>
<tr>
<td>100 &lt; s ≤ 105</td>
<td>16</td>
</tr>
<tr>
<td>105 &lt; s ≤ 110</td>
<td>18</td>
</tr>
<tr>
<td>110 &lt; s ≤ 120</td>
<td>22</td>
</tr>
<tr>
<td>120 &lt; s ≤ 140</td>
<td>12</td>
</tr>
</tbody>
</table>

(a) On the grid, draw a histogram for the information in the table.

(b) Find an estimate for the median.

...................................................... km/h

(Total for Question 14 is 5 marks)
15 Show that \( \frac{a}{b+1} - \frac{a}{(b+1)^2} \) can be written as \( \frac{ab}{(b+1)^2} \).

(Total for Question 15 is 2 marks)

16 The diagram shows a sector of a circle of radius 4 cm.

![Diagram of a sector of a circle]

Work out the length of the arc \( ABC \).
Give your answer correct to 3 significant figures.

...................................................... cm

(Total for Question 16 is 2 marks)
17 The product of two consecutive positive integers is added to the larger of the two integers.

Prove that the result is always a square number.

(Total for Question 17 is 3 marks)
18 Here is a speed-time graph for a car.

(a) Work out an estimate for the distance the car travelled in the first 10 seconds. Use 5 strips of equal width.

....................................................... m

(3)

(b) Is your answer to (a) an underestimate or an overestimate of the actual distance? Give a reason for your answer.

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(1)

(Total for Question 18 is 4 marks)
19 Prove algebraically that the recurring decimal $0.31\dot{8}$ can be written as $\frac{7}{22}$.

(Total for Question 19 is 2 marks)
20

\[\text{OAB is a triangle.}\]
\[\text{P is the point on AB such that } AP : PB = 5 : 3\]

\[\overrightarrow{OA} = 2a\]
\[\overrightarrow{OB} = 2b\]
\[\overrightarrow{OP} = k(3a + 5b) \text{ where } k \text{ is a scalar quantity.}\]

Find the value of \(k\).
21

$ABC$ is a triangle.
$D$ is a point on $AB$.

Work out the area of triangle $BCD$.
Give your answer correct to 3 significant figures.

\[ \text{Area of } \triangle BCD = \text{cm}^2 \]

(Total for Question 21 is 5 marks)
22 There are $y$ black socks and 5 white socks in a drawer.

Joshua takes at random two socks from the drawer.

The probability that Joshua takes one white sock and one black sock is \( \frac{6}{11} \)

(a) Show that \( 3y^2 - 28y + 60 = 0 \)

(b) Find the probability that Joshua takes two black socks.

(Total for Question 22 is 7 marks)
23 (a) Write \(2x^2 + 16x + 35\) in the form \(a(x + b)^2 + c\) where \(a\), \(b\), and \(c\) are integers.

(b) Hence, or otherwise, write down the coordinates of the turning point of the graph of \(y = 2x^2 + 16x + 35\)

(Total for Question 23 is 4 marks)