Instructions

• Use black ink or ball-point pen.
• Fill in the boxes at the top of this page with your name, centre number and candidate number.
• Answer all questions.
• Answer the questions in the spaces provided
  – there may be more space than you need.
• Calculators may be used.
• If your calculator does not have a $\pi$ button, take the value of $\pi$ to be 3.142 unless the question instructs otherwise.
• Diagrams are NOT accurately drawn, unless otherwise indicated.
• You must show all your working out.

Information

• The total mark for this paper is 80
• The marks for each question are shown in brackets
  – use this as a guide as to how much time to spend on each question.

Advice

• Read each question carefully before you start to answer it.
• Keep an eye on the time.
• Try to answer every question.
• Check your answers if you have time at the end.
Answer ALL questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 Frank, Mary and Seth shared some sweets in the ratio 4 : 5 : 7
Seth got 18 more sweets than Frank.

Work out the total number of sweets they shared.

\[
\begin{align*}
F : M : S & \quad x \quad \frac{4}{7} \\
4 : 5 : 7 & \quad \frac{4}{5} = \frac{x}{y} \\
x : y : x+18 & \quad 7x = 4x + 72 \\
24 : 30 : 42 & \quad 3x = 72 \\
& \quad x = 24 \\
24 + 30 + 42 & = 96
\end{align*}
\]

(Total for Question 1 is 3 marks)

2 \(PQR\) is a right-angled triangle.

\[
\begin{align*}
\sin \angle P & = \frac{5}{14} \\
x & = \sin^{-1} \left( \frac{5}{14} \right) \\
x & = 20.9^\circ \text{(1dp)}
\end{align*}
\]

(Total for Question 2 is 2 marks)
3. Here are the first four terms of an arithmetic sequence. 

\[6, 10, 14, 18\]

4 is the common difference!

(a) Write an expression, in terms of \(n\), for the \(n\)th term of this sequence.

\[
\text{add 4 to a term each time. To Find } 2, \text{ we subtract 4 from 6 the first term}
\]

\[4n + 2\]

Since the common difference is 4, we multiply 4 by \(n\)

\[= 4n + ? \quad : \quad 6 - 4 = 2\]

\[4n + 2\]

The \(n\)th term of a different arithmetic sequence is \(3n + 5\)

(b) Is 108 a term of this sequence?

Show how you get your answer.

\[\text{Solving } 3n + 5 = 108\]

\[3n = 108 - 5\]

\[3n = 103\]

\[n = \frac{103}{3} = 34 \frac{1}{3}\]

Hence, 108 is not a member of the sequence because it does not give \(n\) a whole number

(Total for Question 3 is 4 marks)
4. Axcl and Lethna are driving along a motorway.

They see a road sign.
The road sign shows the distance to Junction 8
It also shows the average time drivers take to get to Junction 8

\[ S = \frac{D}{T} \]

To Junction 8
30 miles
26 minutes

The speed limit on the motorway is 70 mph.

Lethna says

"We will have to drive faster than the speed limit to drive 30 miles in 26 minutes."

Is Lethna right?
You must show how you get your answer.

\[ \frac{70}{T} = \frac{30}{\frac{3}{7}} \]

\[ T = \frac{30 \times \frac{7}{3}}{70} = \frac{3}{7} \text{ hrs} \]

\[ = 25 \text{ min } 43 \text{ sec} \]

Hence Lethna is wrong, it will take under 26 minutes to drive 30 miles at 70 mph.

(Total for Question 4 is 3 marks)
The table shows some information about the foot lengths of 40 adults.

<table>
<thead>
<tr>
<th>Foot length (f cm)</th>
<th>Number of adults</th>
<th>Mid Point</th>
<th>F x M.P</th>
</tr>
</thead>
<tbody>
<tr>
<td>16 ≤ f &lt; 18</td>
<td>3</td>
<td>17</td>
<td>51</td>
</tr>
<tr>
<td>18 ≤ f &lt; 20</td>
<td>6</td>
<td>19</td>
<td>114</td>
</tr>
<tr>
<td>20 ≤ f &lt; 22</td>
<td>10</td>
<td>21</td>
<td>210</td>
</tr>
<tr>
<td>22 ≤ f &lt; 24</td>
<td>12</td>
<td>23</td>
<td>276</td>
</tr>
<tr>
<td>24 ≤ f &lt; 26</td>
<td>9</td>
<td>25</td>
<td>225</td>
</tr>
<tr>
<td><strong>Totals</strong></td>
<td><strong>40</strong></td>
<td><strong>876</strong></td>
<td></td>
</tr>
</tbody>
</table>

(a) Write down the modal class interval.

22 ≤ f < 24

(b) Calculate an estimate for the mean foot length.

\[
\text{Estimate mean} = \frac{876}{40} = 21.9
\]

\[21.9\text{ cm}\]

(Total for Question 5 is 4 marks)
Triangles $ABD$ and $BCD$ are right-angled triangles.

Work out the value of $x$.
Give your answer correct to 2 decimal places.

We first find $BD$

$AD^2 - AB^2 = BD^2$

$BD^2 = 100 - 25$

$BD = \sqrt{75}$

$x^2 = (\sqrt{75})^2 + 4^2$

$x = \sqrt{91} = 9.54\text{cm}$
7. The graph of \( y = f(x) \) is drawn on the grid.

(a) Write down the coordinates of the turning point of the graph.

\[ (1, 4) \]  

(b) Write down the roots of \( f(x) = 2 \)

\[ -0.4, 2.4 \]  

(c) Write down the value of \( f(0.5) \)

\[ 3.75 \]  

(Total for Question 7 is 3 marks)
8 In a box of pens, there are
three times as many red pens as green pens
and two times as many green pens as blue pens.

For the pens in the box, write down
the ratio of the number of red pens to the number of green pens to the number of blue pens.

\[
\begin{align*}
R : G : B &= 3x : x : \frac{x}{2} \\
3 : 1 : 0.5 &= 6 : 2 : 1
\end{align*}
\]

(Total for Question 8 is 2 marks)
9 \hspace{1em} ABCD \text{ is a rectangle.} \\
EFGH \text{ is a trapezium.}

All measurements are in centimetres. \\
The perimeters of these two shapes are the same.

Work out the area of the rectangle.

\[
2(3x + 4) + 2(4x) = x + 7x - 3 + 2(5x)
\]

\[
6x + 8 + 8x = 8x - 6 + 10x
\]

\[
6x + 8 = 10x - 6
\]

\[
14 = 4x
\]

\[
x = \frac{14}{4} = 3.5
\]

\[
\text{Length} = 3(3.5) + 4 = 14.5
\]

\[
\text{Width} = 4(3.5) = 14
\]

\[
\text{Area} = 14 \times 14.5 = 203 \text{ cm}^2
\]

(Total for Question 9 is 5 marks)
Katy invests £2000 in a savings account for 3 years. The account pays compound interest at an annual rate of

2.5% for the first year
x% for the second year
x% for the third year

There is a total amount of £2124.46 in the savings account at the end of 3 years.

(a) Work out the rate of interest in the second year.

After year 1: $2000 \times 1.025 = £2050$

After 3 yrs:

$$2050 \times x^2 = 2124.46$$

$$\sqrt{(1 + \frac{x}{100})^2 \times x^2} = \sqrt{1.036321951}$$

$$1 + \frac{x}{100} = 1.01799899$$

$$\frac{x}{100} = 0.01799899 \times 100$$

$$x = 1.8\% \quad (1dp)$$

Katy goes to work by train.

The cost of her weekly train ticket increases by 12.5% to £225

(b) Work out the cost of her weekly train ticket before this increase.

$$12.5\% = 225 \div 112.5$$

$$\div 112.5$$

$$1\% = 2$$

$$\frac{2}{100} = 2 \div 200.00$$

£200.00

(Total for Question 10 is 6 marks)
$S$ and $T$ are points on the circumference of a circle, centre $O$.
$PT$ is a tangent to the circle.
$SOP$ is a straight line.
Angle $OPT = 32^\circ$

Work out the size of the angle marked $x$.
You must give a reason for each stage of your working.

\[
\angle OTP = 90^\circ \text{ Tangent meets radius at } 90^\circ \\
\angle POT = 58^\circ \text{ Angles in a triangle add up to } 180^\circ \\
\angle SOT = 122^\circ \text{ Angles on a straight line add up to } 180^\circ \\
\]

\[x = 29^\circ \frac{(180 - 122)}{2} \text{ Base angles of an isosceles triangle are equal.}\]
A and B are two sets of traffic lights on a road.

The probability that a car is stopped by lights A is 0.4

If a car is stopped by lights A, then the probability that the car is *not* stopped by lights B is 0.7

If a car is *not* stopped by lights A, then the probability that the car is *not* stopped by lights B is 0.2

(a) Complete the probability tree diagram for this information.

```
lights A                        lights B
  stop                          stop   0.12
    0.4                           0.3
  not stop                       0.7
    0.6                           0.2
```

Mark drove along this road.
He was stopped by just one of the sets of traffic lights.

(b) Is it more likely that he was stopped by lights A or by lights B?
You must show your working.

It is more likely to be lights B.

\[ P(\text{not stop, stop}) = 0.48 \]

\[ P(\text{stop, not stop}) = 0.28 \]

\[ 0.48 > 0.28 \text{ so more chance of being stopped at B.} \]
13 \( d \) is inversely proportional to \( c \)

When \( c = 280, \ d = 25 \)

Find the value of \( d \) when \( c = 350 \)

\[
\begin{align*}
\frac{d}{c} &= \frac{1}{c} \\
\therefore \quad d &= \frac{k}{c} \\
\text{Hence } k &= d \times c \\
&= 25 \times 280 \\
&= 7000
\end{align*}
\]

\[
\begin{align*}
\frac{d}{c} &= \frac{1}{c} \\
d &= \frac{7000}{350} \\
&= 20
\end{align*}
\]

Note
To change the proportional sign to an equal sign, we introduce a constant \( k \)

\[d = 20\]

(Total for Question 13 is 3 marks)

14 Prove algebraically that

\[(2n + 1)^2 - (2n + 1)\]

is an even number

for all positive integer values of \( n \).

\[
\begin{align*}
(2n+1)^2 &- (2n+1) \\
4n^2 + 4n + 1 &- (2n + 1) \\
4n^2 + 4n + 1 - 2n - 1 &\text{ collect the like terms together} \\
4n^2 + 2n &\text{ Any number (integer) multiplied by 2 is even.} \\
2(2n^2 + n) &
\end{align*}
\]

(Total for Question 14 is 3 marks)
15 Prove algebraically that the recurring decimal \(0.2\overline{5}\) has the value \(\frac{23}{90}\)

\[
x = 0.25
\]

\[
10x = 2.555\ldots \quad (i)
\]

\[
100x = 25.555\ldots \quad (ii)
\]

\[
(ii) - (i) \quad (25.555\ldots - 2.5) = 23
\]

\[
90x = 23
\]

\[
\therefore \quad x = \frac{23}{90}
\]

(Total for Question 15 is 2 marks)

16 Show that \(\frac{1}{6x^2 + 7x - 5} + \frac{1}{4x^2 - 1}\) simplifies to \(\frac{ax + b}{cx + d}\) where \(a, b, c\) and \(d\) are integers.

\[
\frac{1}{6x^2 + 7x - 5} \times \frac{4x^2 - 1}{1}
\]

Fup the divisor when changing the \(\div\) sign to \(\times\)

\[
\frac{4x^2 - 1}{6x^2 + 7x - 5}
\]

Factorise the numerator and denominator separately

\[
\frac{(2x + 1)(2x - 1)}{(2x - 1)(3x + 5)} \rightarrow\text{Difference of two squares}
\]

\[
\frac{(2x - 1)(3x + 5)}{2x + 1}
\]

\[
= \frac{2x + 1}{3x + 5}
\]

(Total for Question 16 is 3 marks)
17 The diagram shows a sector of a circle of radius 7 cm. Work out the length of arc $AB$. Give your answer correct to 3 significant figures.

$$\text{Arc Length} = \frac{\theta}{360} \times 2\pi r$$

$$= \frac{40}{360} \times 2\pi r \quad \Rightarrow \quad 4.88692190558 \quad = 4.89 \text{ cm (3 sig)}$$

(Total for Question 17 is 2 marks)
\[ m = \frac{\sqrt{s}}{t} \quad s = 3.47 \text{ correct to 3 significant figures} \]
\[ t = 8.132 \text{ correct to 4 significant figures} \]

By considering bounds, work out the value of \( m \) to a suitable degree of accuracy.
Give a reason for your answer.

Upper \( m \):
\[ \frac{\sqrt{s}}{t} = \frac{3.475}{8.1315} = 0.2292486243 \]

Lower \( m \):
\[ \frac{\sqrt{s}}{t} = \frac{3.465}{8.1325} = 0.2288903839 \]

\[ m = 0.229 (3 \text{sig}) \]

Upper and lower bound both round to 0.229 to 3 sig

(Total for Question 18 is 5 marks)
19 The graph of $y = f(x)$ is shown on both grids below.

(a) On the grid above, sketch the graph of $y = f(-x)$

(b) On this grid, sketch the graph of $y = -f(x) + 3$  

(Total for Question 19 is 2 marks)
Solve algebraically the simultaneous equations

\[ x^2 + y^2 = 25 \]
\[ y - 2x = 5 \]

**Make \( y \) the subject:**

\[ y = 5 + 2x \]

\[ x^2 + (2x + 5)^2 = 25 \]
\[ x^2 + 4x^2 + 20x + 25 = 25 \]
\[ 5x^2 + 20x = 0 \]
\[ 5x(x + 4) = 0 \]
\[ 5x = 0 \text{ and } x + 4 = 0 \]
\[ x = 0 \text{ or } x = -4 \]

When \( x = 0 \) and \( x + 4 = -4 \), what are the respective values of \( y \)?

\[ y = 5 \]
\[ y = -3 \]

\[ x = 0 \]
\[ y = 5 \]

\[ x = -4 \]
\[ y = -3 \]

\[ x = 0, y = 5 \text{ or } x = -4, y = -3 \]

(Total for Question 20 is 5 marks)
21 In triangle $RPQ$,

\[ RP = 8.7 \text{ cm} \]
\[ PQ = 5.2 \text{ cm} \]
\[ \text{Angle } PRQ = 32^\circ \]

(a) Assuming that angle $PQR$ is an acute angle,
calculate the area of triangle $RPQ$.
Give your answer correct to 3 significant figures.

\[ \frac{\sin x}{8.7} = \frac{\sin 32^\circ}{5.2} \]
\[ \sin x = \frac{\sin 32^\circ \times 8.7}{5.2} \]
\[ x = \sin^{-1}\left(\frac{\sin 32^\circ \times 8.7}{5.2}\right) \]
\[ x = 62.44^\circ \]

\[ \angle RPQ = 180 - 32 - 62.44 = 85.55^\circ \]

Area of triangle $RPQ = \frac{1}{2} \times ab \times \sin C$
\[ = \frac{1}{2} \times 8.7 \times 5.2 \times \sin 85.55^\circ \]
\[ = 22.6 \text{ cm}^2 \text{ (3 sig)} \] (4)

(b) If you did not know that angle $PQR$ is an acute angle, what effect would this have on
your calculation of the area of triangle $RPQ$?

If $PQR$ could have been obtuse, then we would
have a different triangle to find the area
of.

(Total for Question 21 is 5 marks)
22 A frustum is made by removing a small cone from a large cone as shown in the diagram.

The frustum is made from glass.
The glass has a density of 2.5 g/cm³

Work out the mass of the frustum.
Give your answer to an appropriate degree of accuracy.

Volume of Frustum = \[ \frac{1}{3} \pi r^2 \cdot \frac{h}{3} \cdot \frac{h}{3} \cdot \frac{h}{3} \]

\[ = \frac{1}{3} \times \pi \times 6^2 \times 15 \]

\[ = 565.48 \text{ cm}^3 \]

\[ = 565.48 \times 10^{-3} \text{ m}^3 \]

M = D \times V

\[ = 2.5 \text{ g/cm}^3 \times 544.54272 \]

\[ = 1361.356816569 \text{g} \]

\[ = 1361 \text{ g} \]

(Total for Question 22 is 5 marks)

TOTAL FOR PAPER IS 80 MARKS